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Gauge Unification, Yukawa Hierarchy and the μ Problem

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The hierarchy in the Yukawa couplings may be the result of a gauged horizontal $U(1)_H$ symmetry. If the mixed anomalies of the Standard Model gauge group with $U(1)_H$ are cancelled by a Green-Schwarz mechanism, a relation between the gauge couplings, the Yukawa couplings and the μ-term arises. Assuming that at a high energy scale $g_3^2 = g_2^2 = \frac{5}{3}g_1^2$ and $(m_e m_\mu m_\tau)/(m_d m_s m_b) \sim \lambda$ (where λ is of the order of the Cabibbo angle), the $U(1)_H$ symmetry solves the μ-problem with $\mu \sim \lambda m_{3/2}$.

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A possible intriguing relation between hierarchies in the fermion Yukawa couplings and unification of gauge couplings has been recently proposed by Binétruy and Ramond [BR] [1] and by Ibáñez and Ross [2]. The relation arises if anomalies of a gauged horizontal $U(1)_H$ symmetry are cancelled by a Green-Schwarz mechanism [3]. One of the basic assumptions of BR was that the μ-term in the superpotential is neutral under $U(1)_H$. In this work, we investigate the more general case, namely that the μ-term carries an arbitrary charge. We point out that if we take that, at a high scale, (a) the gauge couplings satisfy

$$g_3^2 = g_2^2 = \frac{5}{3} g_1^2, \tag{1}$$

and (b) the fermion masses satisfy

$$\frac{m_e m_\mu m_\tau}{m_d m_s m_b} \sim \lambda \tag{2}$$

(where $\lambda \sim 0.2$ is the small breaking parameter of $U(1)_H$), then the horizontal symmetry solves the μ-problem with

$$\mu \sim \lambda m_{3/2}. \tag{3}$$

We work in the framework of Supersymmetry and $SU(3)_C \times SU(2)_L \times U(1)_Y \times U(1)_H$ gauge symmetry. We now list the assumptions that lead to our results. (We also show that some of the assumptions made by BR are actually not necessary for their result.)

1. *The smallness and hierarchy among the Yukawa couplings is a result of a horizontal $U(1)_H$ symmetry that is broken by a single small parameter $\lambda (\sim 0.2)$ whose charge under $U(1)_H$ is defined to be –1.*[1]

It follows that the order of magnitude of the various Yukawa couplings,

$$\mathcal{L}^{\mathrm{Y}} = \lambda^u_{ij} Q_i \bar{u}_j \phi_u + \lambda^d_{ij} Q_i \bar{d}_j \phi_d + \lambda^\ell_{ij} L_i \bar{\ell}_j \phi_d, \tag{4}$$

[1] The parameter λ is the ratio between the VEV of a SM-singlet scalar field that carries charge –1 under $U(1)_H$ and a somewhat higher scale where the information about $U(1)_H$ breaking is communicated to the light fermions [4].

can be estimated by the following selection rules [5]:

$$\begin{aligned}
\lambda^u_{ij} &\sim \begin{cases} \lambda^{H(Q_i)+H(\bar u_j)+H(\phi_u)} & H(Q_i)+H(\bar u_j)+H(\phi_u)\geq 0, \\ 0 & H(Q_i)+H(\bar u_j)+H(\phi_u)<0, \end{cases} \\
\lambda^d_{ij} &\sim \begin{cases} \lambda^{H(Q_i)+H(\bar d_j)+H(\phi_d)} & H(Q_i)+H(\bar d_j)+H(\phi_d)\geq 0, \\ 0 & H(Q_i)+H(\bar d_j)+H(\phi_d)<0, \end{cases} \\
\lambda^\ell_{ij} &\sim \begin{cases} \lambda^{H(L_i)+H(\bar \ell_j)+H(\phi_d)} & H(L_i)+H(\bar \ell_j)+H(\phi_d)\geq 0, \\ 0 & H(L_i)+H(\bar \ell_j)+H(\phi_d)<0. \end{cases}
\end{aligned} \tag{5}$$

The zeros, corresponding to negative charges, are a result of the holomorphy of the superpotential [5]. If there are no zero eigenvalues (as experimentally known for quarks and leptons, with the possible exception of $m_u=0$ [6]), we get

$$\begin{aligned}
\det M^u &\sim \langle\phi_u\rangle^3\lambda^{\sum_i[H(Q_i)+H(\bar u_i)]+3H(\phi_u)}, \\
\det M^d &\sim \langle\phi_d\rangle^3\lambda^{\sum_i[H(Q_i)+H(\bar d_i)]+3H(\phi_d)}, \\
\det M^\ell &\sim \langle\phi_d\rangle^3\ \lambda^{\sum_i[H(L_i)+H(\bar \ell_i)]+3H(\phi_d)}.
\end{aligned} \tag{6}$$

Note that (6) requires neither that third generation fermions acquire masses without horizontal suppression, nor that all excess charges in λ^f_{ij} are positive (two assumptions made by BR). It only requires that all fermion masses are non-zero, and so it holds in any phenomenologically acceptable model.

2. *The only fields that are in chiral representations of $U(1)_H$ and transform non-trivially under the SM gauge group are the MSSM supermultiplets.*[2]

This allows one to calculate the mixed anomalies C_n in $SU(n)^2\times U(1)_H$, (in $n=3,2,1$ we refer to the $SU(3)_C\times SU(2)_L\times U(1)_Y$ factors in the SM gauge group) in terms of the H-charges carried by the MSSM fields only. Explicitly [1]:

$$\begin{aligned}
C_3 &= \sum_i[2H(Q_i)+H(\bar u_i)+H(\bar d_i)], \\
C_2 &= \sum_i[3H(Q_i)+H(L_i)]+H(\phi_u)+H(\phi_d), \\
C_1 &= \sum_i[\frac{1}{3}H(Q_i)+\frac{8}{3}H(\bar u_i)+\frac{2}{3}H(\bar d_i)+H(L_i)+2H(\bar\ell_i)]+H(\phi_u)+H(\phi_d).
\end{aligned} \tag{7}$$

[2] The fields required by the Froggatt-Nielsen mechanism [4] are either SM-singlets or in vector representations, consistent with this assumption.

Eqs. (6) and (7) lead to two independent relations between the anomalies C_i, the determinants of the fermion mass matrices, the VEVs of the Higgs doublets and the sum of Higgs charges

$$H(\phi) \equiv H(\phi_d) + H(\phi_u). \tag{8}$$

These are

$$(\det M^u)(\det M^d) \sim \langle\phi_u\rangle^3\langle\phi_d\rangle^3\lambda^{C_3+3H(\phi)}, \tag{9}$$

$$(\det M^\ell)(\det M^u)^{\frac{4}{3}}(\det M^d)^{\frac{1}{3}} \sim \langle\phi_u\rangle^4\langle\phi_d\rangle^4\lambda^{\frac{1}{2}(C_1+C_2)+3H(\phi)}. \tag{10}$$

The reason that there are only two independent relations is that the Yukawa couplings have an accidental $U(1)_B \times U(1)_L$ symmetry (B and L stand here for Baryon and Lepton number, respectively); only C_3 and $C_1 + C_2$ are $U(1)_B \times U(1)_L$ invariant.

The Yukawa sector has yet another accidental symmetry, $U(1)_X$, with $X(\phi_d) = -X(\bar{d}_i) = -X(\bar{\ell}_i)$ and all other supermultiplets carrying $X = 0$. One combination of the anomalies is $U(1)_X$ invariant and should, therefore, appear in an $H(\phi)$-independent relation. Indeed, dividing (10) by (9), we get

$$\frac{(\det M^\ell)(\det M^u)^{1/3}}{(\det M^d)^{2/3}} \sim \langle\phi_u\rangle\langle\phi_d\rangle\lambda^{(C_1+C_2-2C_3)/2}. \tag{11}$$

The LHS of this relation can be estimated to be $\lesssim \mathcal{O}(m_s m_c)$ (assuming approximate geometrical hierarchies and $\det M^\ell \lesssim \det M^d$), while the RHS is (for $\tan\beta \lesssim m_t/m_b$) $\gtrsim \mathcal{O}(m_b m_t)\lambda^{(C_1+C_2-2C_3)/2}$. The conclusion is then that $C_1 + C_2 - 2C_3 > 10$ (for $\lambda \sim 0.2$): the mixed anomalies cannot vanish simultaneously. This conclusion is independent of $H(\phi)$. If $U(1)_H$ is a local symmetry, there should exist a mechanism to cancel these anomalies.

3. *The mixed anomalies are cancelled by a Green-Schwarz mechanism* [7], *with Im(S) playing the role of the $U(1)_H$ axion (S stands for the dilaton supermultiplet).*

This is possible only if [3]

$$\frac{C_1}{k_1} = \frac{C_2}{k_2} = \frac{C_3}{k_3} = \delta_{GS}, \tag{12}$$

where k_i are the Kac-Moody levels and δ_{GS} is a constant that gives the transformation law of the axion under $U(1)_H$. In general, k_2 and k_3 are integers and k_1 is a rational number.

The gauge couplings for the SM gauge group are given (at the string scale) by

$$g_i^2 = \frac{g^2}{k_i}. \tag{13}$$

We can then relate the mixed anomalies to the gauge couplings:

$$C_i = \frac{g^2 \delta_{GS}}{g_i^2}. \tag{14}$$

We next turn our attention to another relation that results from (9) and (10) and is independent of $\langle \phi_u \rangle$, $\langle \phi_d \rangle$:

$$\frac{\det M^d}{\det M^\ell} \sim \lambda^{-\frac{1}{2}[C_1 + C_2 - \frac{8}{3}C_3] + H(\phi)}. \tag{15}$$

Using (14) we can rewrite (15) as

$$\frac{\det M^d}{\det M^\ell} \sim \lambda^{-\frac{g^2 \delta_{GS}}{2}\left[\frac{1}{g_1^2} + \frac{1}{g_2^2} - \frac{8}{3g_3^2}\right] + H(\phi)}. \tag{16}$$

Before proceeding with the analysis of (16), we would like to discuss the relation between $H(\phi)$ and the μ-problem. $H(\phi)$ is the horizontal charge carried by the μ term in the superpotential,

$$\mu \phi_u \phi_d. \tag{17}$$

The μ-term is special in that it can come from two sources: it may arise from the high energy superpotential, in which case it is holomorphic and its natural scale is unknown (but most likely $\sim M_{Pl}$); or it may arise from the Kähler potential, in which case it is not holomorphic and its natural scale is the SUSY breaking scale [8]. In other words, the selection rule for μ is

$$\mu \sim \begin{cases} \tilde{\mu} \lambda^{H(\phi)} & H(\phi) \geq 0, \\ m_{3/2} \lambda^{|H(\phi)|} & H(\phi) < 0, \end{cases} \tag{18}$$

where $\tilde{\mu}$ is the unknown natural scale of μ (most likely $\tilde{\mu} \sim M_{Pl}$). This suggests that the horizontal symmetry may actually solve the μ-problem: if the charge carried by the μ-term is negative, then it cannot come from the superpotential and its natural scale is $m_{3/2}$. Note, however, that the charge cannot be too negative, because $\mu \ll m_{3/2}$ is phenomenologically unacceptable. With $\lambda \sim 0.2$, we can allow $H(\phi) = -1$, which would give $\mu / m_{3/2} \sim \lambda$. We

conclude that a continuous horizontal symmetry can solve the μ-problem. The solution requires an almost unique choice of charge, namely that the μ-term carries one negative unit of the horizontal charge.

Eq. (18) gives a new interpretation of (16); it is a relation between Yukawa couplings, the gauge couplings and the μ-term. For example, if $H(\phi) < 0$, (16) can be rewritten as

$$\frac{\mu}{m_{3/2}} \frac{\det M^d}{\det M^\ell} \sim \lambda^{-\frac{g^2 \delta_{GS}}{2}\left[\frac{1}{g_1^2}+\frac{1}{g_2^2}-\frac{8}{3g_3^2}\right]}. \tag{19}$$

BR made the following further three assumptions:

(i) $H(\phi) = 0$.

(ii) $k_2 = k_3$, namely $g_2 = g_3$.

(iii) $m_e m_\mu m_\tau \sim m_d m_s m_b$ (at the high scale).

Then (16) leads to the interesting result $\sin^2\theta_W \equiv \frac{g_1^2}{g_1^2+g_2^2} = \frac{3}{8}$, in perfect agreement with extrapolated phenomenology. The weakest assumption in this derivation is that of $H(\phi) = 0$ which is just the rather arbitrary ansatz that μ is unsuppressed relative to $\tilde{\mu}$ (which is unknown).

The way in which we will proceed is to use the extrapolated phenomenology to make assumptions about the gauge couplings and the Yukawa couplings and find the consequences for $H(\phi)$ and the μ-term. Specifically, we make the following assumptions, which are based on running the various gauge and Yukawa couplings to the high scale:

4. *At the string scale, the gauge couplings satisfy*

$$g_3^2 = g_2^2 = \frac{5}{3} g_1^2. \tag{20}$$

5. *At the string scale, the fermion masses satisfy*

$$\frac{m_e}{m_\mu} \sim \lambda^3, \quad \frac{m_\mu}{m_\tau} \sim \lambda^2, \quad \frac{m_d}{m_s} \sim \lambda^2, \quad \frac{m_s}{m_b} \sim \lambda^2. \tag{21}$$

(Here, we differ from BR; they take $\frac{m_e}{m_\mu} \sim \lambda^2$.) Consequently, if at the high scale m_b and m_τ approximately unify, namely $m_\tau \sim m_b$, we get

$$\frac{m_e m_\mu m_\tau}{m_d m_s m_b} \sim \lambda. \tag{22}$$

This leads, through eq. (16), to

$$H(\phi) = -1, \tag{23}$$

which is just the right value to solve the μ-problem (see (19))

$$\mu \sim \lambda m_{3/2}. \tag{24}$$

To summarize: if anomalies in a gauged horizontal $U(1)$ symmetry are cancelled by a GS mechanism, then interesting relations among gauge couplings, Yukawa couplings and the μ term arise. The values of the gauge and Yukawa couplings, when extrapolated from their measured low energy values, imply that the scale of the μ term is below (but not far below) the SUSY breaking scale.

While this paper was in writing, we received a preprint by Dudas, Pokorski and Savoy [9] that also investigates the general $H(\phi)$ case.

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